Properties of wide-peak autosolitons in electron-hole and gas plasmas

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With the use of the multifunctional variational method we have analyzed the shape and other parameters of a wide-peak autosoliton, which can be excited in electron-hole or slightly ionized gas plasmas heated by constant electric current or electromagnetic radiation. We have shown that this autosoliton exists even in very weakly nonequilibrated plasmas. In a number of cases the minimal pumping power needed to maintain this autosoliton is proportional to ϵ^2, whereas the maximum temperature of hot carriers in its center, $T_{\max} \propto \epsilon^{-4}$, where $\epsilon = l/L \ll 1$, L is the ambipolar diffusion length, and l is the length of the hot electron energy relaxation.

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As a result of the instability of the homogeneous state in highly nonequilibrated systems, different kinds of patterns may spontaneously appear [1–4]. Inside the stability region of the homogeneous state in such systems one can excite solitary patterns — autosolitons (AS) [3,5]. Autosolitons in the form of strongly nonequilibrated localized regions can be easily induced in slightly ionized gas or semiconductor plasmas heated by the constant electric current or electromagnetic radiation (Joule heating), if certain conditions are fulfilled [3,5,6]. Depending on the parameters, either wide AS with the size of the order of ambipolar diffusion length L, or narrow peak AS with the size of the order of hot electron energy relaxation length l may, as a rule, form in it. General qualitative theory of such AS is presented in [3,5].

At the same time, under rather ordinary conditions, AS of a completely different type may form in electron-hole or gas plasma. These AS — wide-peak autosolitons — have enormously high amplitude and the size bigger than the ambipolar diffusion length L. Wide-peak AS is a striking phenomenon: it is a strongly nonequilibrated region of big size forming in slightly nonequilibrated plasma. In this sense, it is similar to the phenomenon of ball lightning in the atmosphere. The possibility of existence for such strongly nonequilibrated regions was predicted from the qualitative considerations [7]. Wide-peak AS were found in numerical simulations and experimental investigations of the photogenerated plasma in Ge [8–11].

Wide-peak AS cannot be described within the existing approaches, as was emphasized in review [5]. Moreover, up to now there has been no theory of these AS, and only a few of their qualitative properties were investigated numerically [9,10]. In this paper, we analytically construct and study a solution in the form of wide-peak AS. For this purpose, we employ the ideas of the multifunctional variational method, developed in Ref. [12].

For definiteness let us consider nondegenerate symmetric electron-hole plasma (EHP) heated by constant electric current, in which concentrations, as well as the other parameters, of electrons and holes are equal to each other: $n = p$, effective masses $m_e = m_h$, relaxation times of momentum $\tau_{pe} = \tau_{ph} = \tau_p \propto E^p$ and energy $\tau_{\epsilon e} = \tau_{\epsilon h} = \tau_\epsilon \propto E^s$, where E is the energy of an electron; the diffusion coefficients of electrons and holes $D_e = D_h = D$, and recombination times $\tau_{re} = \tau_{rh} = \tau_r$. Without loss of generality we will assume that τ_r is constant. Let us also assume that the concentration n is such that the conditions of quasineutrality and the condition $\tau_p \ll \tau_{ee} \ll \tau_\epsilon$, where τ_{ee} is the characteristic interelectron collision time, are fulfilled. This EHP is described by the two balance equations: one for the number of particles n and the other for their effective temperature T [13].

As follows from physics considered in detail in [3,7,8,6,5], wide-peak AS is a stratum perpendicular to the direction of current density $\mathbf{j}$. In other words, the considered AS is essentially one dimensional. Taking it into account, we can write the equations describing EHP as [13–15,5]

$$\frac{\partial n}{\partial t} = \frac{\partial^2}{\partial x^2}(nD(T)) - \frac{n - n_0}{\tau_r}, \tag{1}$$

$$\frac{3}{2}\frac{\partial (nT)}{\partial t} = \frac{\partial^2}{\partial x^2}\left[\left(\frac{5}{2} + p\right) nTD(T)\right] - n\frac{T - T_0}{\tau_\epsilon} + \frac{j^2}{2\sigma}, \tag{2}$$

where the x axis is directed along the current, $j = \text{const}$, $\sigma = 2e\mu n$, where $\mu = e\tau_p(T)/m_e$, is plasma conductivity; $n_0 = G\tau_r$, τ_r and G are recombination time and generation rate of electrons and holes, respectively; T_0 is the temperature of the lattice. Introducing the variables $\theta = T/T_0$ and $\eta = nD(T)/n_0D_0$, where $D_0 = D(T_0)$, we can rewrite Eqs. (1) and (2) in the form

$$\frac{\partial(\eta\theta^{-1-p})}{\partial t} = \frac{\partial^2\eta}{\partial x^2} + 1 - \eta\theta^{-1-p}, \tag{3}$$

$$\alpha\frac{\partial(\eta\theta^{-p})}{\partial t} = \epsilon^2\frac{\partial^2}{\partial x^2}(\eta\theta) + \frac{A\theta}{\eta} - \eta\frac{\theta - 1}{\theta^{1+p+s}}, \tag{4}$$

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where length and time are measured in the units of $L = (D_0\tau_r)^{1/2}$ and τ_r, respectively; $A = j^2\tau_\epsilon^0/4\sigma_0 T_0$ is the plasma excitation level; $\epsilon = l/L$, $\alpha = 3\tau_\epsilon^0/2\tau_r$, $l = \left[\left(\frac{5}{2}+p\right) D_0\tau_\epsilon^0\right]^{1/2}$, $\tau_\epsilon^0 = \tau_\epsilon(T_0)$, and $\sigma_0 = \sigma(n_0, T_0)$.

We would emphasize that formation of AS in plasma is in no way related with overheat instability, and may occur even if the relaxation times τ_ϵ and τ_p do not depend on the energy of the electron [14,15,6,5]. So, for simplicity we will consider the case in which all relaxation times are energy independent, i.e., the case $s = p = 0$.

Let us remind you that the formation of AS in EHP is associated with the existence of thermal diffusion, i.e., dependence of the diffusion coefficient on the carrier temperature, which is $D(T) = D_0 T/T_0 = D_0\theta$ in the considered case. Such a thermal diffusion AS forms in the stable homogeneous EHP when $\epsilon \ll 1$ [3,14,15,6,5]. One can easily derive from Eqs. (3) and (4) with $s = p = 0$ that the homogeneous state of the plasma

$$\eta_h = 1 + A, \qquad \theta_h = 1 + A$$

is stable when $A < 1$ [13,15].

Applying to this EHP an additional localized short perturbation of sufficiently large amplitude, one can excite a wide-peak static AS (Fig. 1) when the conditions $\epsilon \ll 1$, $\alpha \ll 1$, and $A < 1$ are satisfied [8]. Numerical simulations show that the qualitative shape of solutions $\theta(x)$ and $\eta(x)$ does not change over a wide range of ϵ and A. This allows us to use the ideas of the multifunctional variational method developed in Ref. [12]. Slightly modifying this method, let us introduce two functionals

$$\Phi_\eta = \int_0^\infty \left[\frac{\eta_x^2}{2} + \frac{\eta^2}{2\theta} - \frac{\eta_h^2}{2\theta_h} - (\eta - \eta_h)\right] dx, \tag{5}$$

$$\Phi_\theta = \int_0^\infty \left[\frac{\epsilon^2\eta^2\theta_x^2}{2} - \frac{\epsilon^2\theta^2\eta\eta_{xx}}{2} + \theta\eta^2 - \eta^2\ln\theta - \frac{A\theta^2}{2} - \theta_h\eta_h^2 + \eta_h^2\ln\theta_h + \frac{A\theta_h^2}{2}\right] dx, \tag{6}$$

where $\theta_x \equiv \frac{\partial\theta}{\partial x}$, $\eta_x \equiv \frac{\partial\eta}{\partial x}$, $\eta_{xx} \equiv \frac{\partial^2\eta}{\partial x^2}$. We use the second functional in the form of Eq. (6) because the nonlinearities in it become especially simple. It can be easily seen that variating Φ_η with respect to η under fixed $\theta(x)$ and variating Φ_θ with respect to θ under fixed $\eta(x)$, and then equating the variations to zero, one can obtain stationary Eqs. (3) and (4), respectively. It means that for the stationary solutions of Eqs. (3) and (4) the change of functional Φ_η is zero with respect to a small arbitrary variation of solution $\eta(x)$ under fixed $\theta(x)$, and the change of functional Φ_θ is zero with respect to a small arbitrary variation of solution $\theta(x)$ under fixed $\eta(x)$.

Before using these properties of the functionals Φ_η and Φ_θ let us discuss the most adequate shape of trial functions for $\theta(x)$ and $\eta(x)$, and make some estimates for the values of $\theta_{\max} = \theta(0)$ and $\eta_{\max} = \eta(0)$. First, let us define the "peak region" as the region $|x| < \lambda$, where λ is the parameter characterizing the width of the distribution $\theta(x)$, namely, the coordinate of the right minimum of the function $\theta(x)$ (see Fig. 1). In the center of the peak region where $\theta \gg \eta$ Eq. (3) for the stationary case becomes

$$\frac{d^2\eta}{dx^2} + 1 = 0. \tag{7}$$

Since the AS is symmetrical with respect to its center, solution of Eq. (7) in the considered region is

$$\eta = \eta_{\max} - \frac{x^2}{2} = \frac{1}{2}(\mathcal{L}^2 - x^2) + \eta_h, \tag{8}$$

where $\mathcal{L}^2 \equiv 2(\eta_{\max} - \eta_h)$. In view of Eq. (8), stationary Eq. (4) can be approximately written near the point $x = 0$ as

$$\epsilon^2\eta_{\max}\theta_{xx} - \epsilon^2\theta_{\max} + A\frac{\theta_{\max}}{\eta_{\max}} - \eta_{\max} = 0. \tag{9}$$

Here the two first terms describe the diffusion of θ, with $\theta_{\max} \gtrsim \eta_{\max}\theta_{xx}$ (the inverse condition would mean that the AS is narrow), while the third and the fourth determine the power delivered to and removed from EHP. As follows from the physics of AS formation [7,8,3,5], all these processes are significant, so all terms in Eq. (9) must have the same order of magnitude. Taking it into account, one can obtain that

$$\theta_{\max} \sim \epsilon^{-4}, \qquad \eta_{\max} \sim \epsilon^{-2}. \tag{10}$$

As will follow from the analysis of the functionals below, these estimations prove to be true.

Outside the peak region ($|x| > \lambda$ in Fig. 1) distribution $\eta(x)$ goes to η_h approximately as $\exp\left[-(1-A)^{-1/2}|x|\right]$. This result follows from the stationary Eqs. (3) and (4) linearized about the homogeneous state with the term by ϵ^2 in Eq. (4) neglected, and allows us to take the following $\eta(x)$ as a trial function:

$$\eta(x) = \begin{cases} \frac{1}{2}(\mathcal{L}^2 - x^2) + \eta_h, & |x| \le \lambda \\ \frac{1}{2}(\mathcal{L}^2 - \lambda^2)\exp\left(-\frac{|x|-\lambda}{\rho}\right) + \eta_h, & |x| \ge \lambda, \end{cases} \tag{11}$$

where $\rho = (1-A)^{1/2}$, and $\mathcal{L}$ is the parameter characterizing the distribution $\eta(x)$. The trial function for $\theta(x)$ will be as follows:

$$\theta(x) = \begin{cases} a\left[1 - \left(\frac{x}{\lambda}\right)^2\right]^m - \theta_1 + \theta_h, & |x| \le \lambda, \\ -\theta_1\exp\left(-\frac{|x|-\lambda}{\rho}\right) + \theta_h, & |x| \ge \lambda. \end{cases} \tag{12}$$

Having chosen the trial functions, we are now able to calculate the integrals in Eqs. (5) and (6). Let us notice that since θ must be greater than one, the value of θ_1 cannot be greater than A, and without loss of accuracy we can put $\theta_1 = 0$. Also, the best agreement of the results obtained with the use of the multifunctional method with those of numerical simulations of Eqs. (3) and (4) is achieved when $m = 4$. Keeping in mind these facts and estimations of Eq. (10), one can see that the main contribution to the integral in Eq. (5) is

$$\Phi_\eta \cong -\frac{\mathcal{L}^2\lambda}{2} + \frac{(\mathcal{L}^2 - \lambda^2)^2}{8\rho(1+A)}, \tag{13}$$

plus irrelevant terms. The variational derivative of the functional Φ_η is now reduced to the partial derivative with respect to $\mathcal{L}$. Equating it to zero, we get

$$0 = \frac{\partial \Phi_\eta}{\partial \mathcal{L}} = -\mathcal{L}\lambda + \frac{(\mathcal{L}^2 - \lambda^2)\mathcal{L}}{2\rho(1+A)}, \tag{14}$$

which for big values of λ gives

$$\mathcal{L} - \lambda = (1+A)(1-A)^{1/2}. \tag{15}$$

In view of Eqs. (10), the main contribution to the integral in Eq. (6) is given by the region $0 < x < \lambda$. Neglecting θ_h and η_h in Eqs. (11) and (12), the term $\eta^2 \ln\theta$ in Eq. (6), and calculating the integral, with $m = 4$ we get

$$\Phi_\theta \cong 0.0045\epsilon^2 a^2 \lambda^3 - 0.15Aa^2\lambda + 0.18\epsilon^2 a^2 \lambda^{-1}\mathcal{L}^4 - 0.018a\lambda^3\mathcal{L}^2 + 0.1a\lambda\mathcal{L}^4 + 0.002a\lambda^5 + 0.01\epsilon^2 a^2 \lambda \mathcal{L}^2. \tag{16}$$

Calculating partial derivatives with respect to λ and a, and taking into account that in the considered approximation the difference between $\mathcal{L}$ and λ given by Eq. (15) is negligible, we can put $\mathcal{L} = \lambda$ after differentiation and obtain that

$$0 = \frac{\partial \Phi_\theta}{\partial \lambda} = 0.057a\lambda^4 - 0.16\epsilon^2 a^2 \lambda^2 - 0.15Aa^2, \tag{17}$$

$$0 = \frac{\partial \Phi_\theta}{\partial a} = 0.085\lambda^5 + 0.39\epsilon^2 a\lambda^3 - 0.3Aa\lambda. \tag{18}$$

Solving this simple set of algebraic equations in λ and a, and summarizing all results, we can conclude that for small ϵ the shape of wide-peak AS is characterized by

$$\theta_{\max} = \frac{A}{210\epsilon^4}, \quad \eta_{\max} = \frac{A}{17\epsilon^2}, \quad \lambda = L\epsilon^{-1}\left(\frac{A}{8.5}\right)^{1/2}. \tag{19}$$

Note that the expressions for $\theta_{\max}$ and $\eta_{\max}$ are in agreement with Eq. (10). We have checked that for $\epsilon < 0.1$ the results given by Eq. (19) differ from those obtained in numerical solution of Eqs. (3) and (4) by no more than 5%.

Equations (19) were derived under the assumption that $\lambda \gg 1$. More accurate analysis, which takes into account Eq. (15), shows that the solution in the form of wide-peak AS abruptly disappears at

$$A = A_b = 50\epsilon^2. \tag{20}$$

This analysis also shows that when $A \gg A_b$, the first two formulas in Eqs. (19) hold with good accuracy in this case, too, and in the last formula one should replace λ by $\mathcal{L}$.

Thus, as follows from Eqs. (19) and (20), when $\epsilon \ll 1$, wide-peak AS exists at extremely low system's excita-

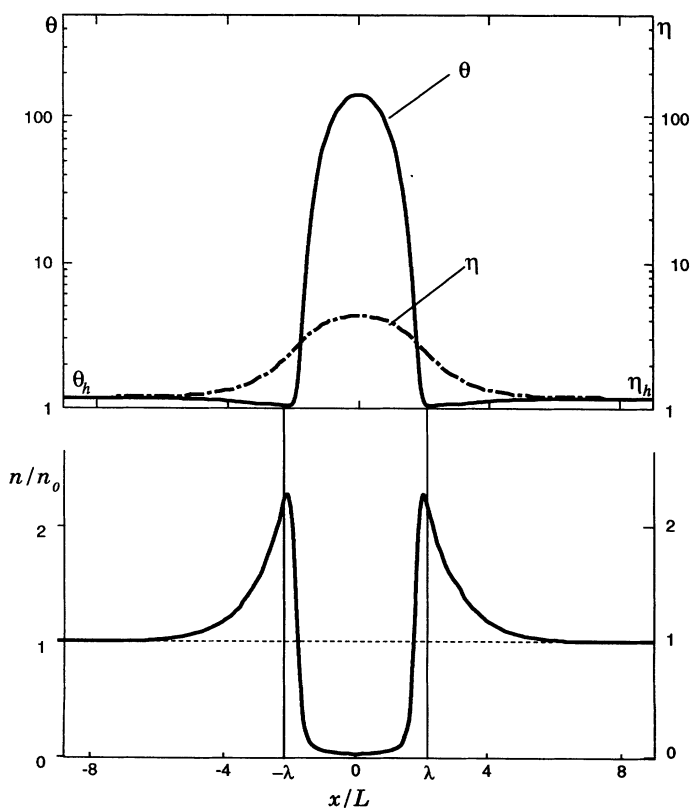

FIG. 1. Distributions of temperature $\theta = T/T_0$, parameter $\eta = nD(T)/n_0D_0$, and concentration n in the form of wide-peak autosoliton. Numerical solutions of Eqs. (3) and (4) for $p = s = 0$, $\epsilon = 0.05$, $\alpha = 0.005$, and $A = 0.195$. Length is measured in units of L.

tion levels, i.e., in practically equilibrium system, and at the same time it has enormously high amplitude and size many times larger than the carriers diffusion length L. The same conclusion can be drawn for the real EHP, parameters of electrons and holes in which do not differ greatly. In this case, one should assume L to be the ambipolar diffusion length. In particular, the same dependences as in Eq. (19) will hold for electron-hole plasma when the relaxation times are such that $p+s=0$ and $p \geq -\frac{1}{2}$. This situation takes place, for example, in polar (PbS, PbTe) and nonpolar (Ge, Si) semiconductors when the momentum and energy of hot electrons are scattered on deformation optical phonons, for which $p=-\frac{1}{2}$, $s=\frac{1}{2}$ [16]. Indeed, for an arbitrary p the functional Φ_η, corresponding to Eq. (3), will be given by Eq. (5) with θ replaced by θ^{1+p}. One can easily see that for $p > -\frac{1}{2}$ and for sufficiently small ϵ and A this functional only weakly depends on the form of distribution $\theta(x)$, so in our approximation the functional Φ_η will remain unchanged for these values of p. Since for $p+s=0$ the functional Φ_θ, corresponding to Eq. (4) has exactly the same form, we can repeat all calculations of Eqs. (13)–(18) and again get Eq. (19) for the AS characteristics. In the case $p=-\frac{1}{2}$, $s=\frac{1}{2}$ one cannot already assume that the term $\eta\theta^{-1-p}$ in Eq. (3) vanishes in the center of AS, so the trial function for $\eta(x)$ inside the peak region should be taken in the form $\eta(x)=\frac{1}{2}(\mathcal{L}^2-b^2x^2)+\eta_h$, where b is of order one [see Eq. (3) at $x=0$ with $\theta_{\max}$ and $\eta_{\max}$ given by Eq. (19) and compare it with Eq. (7)]. Proceeding with the same calculations, we will again arrive at Eq. (19), but with different coefficients. So, for sufficiently small ϵ and A the functional dependence of the AS characteristics on ϵ and A will remain the same as in Eq. (19) for this case, too.

In those semiconductors where the effective mass of holes greatly exceeds the effective mass of electrons, only electrons can be heated. A similar situation is realized in slightly ionized gas plasma heated by electromagnetic radiation. In these cases the ambipolar diffusion coefficient is $D(T)=(T+T_0)\mu_i(T_0)$, where μ_i is the mobility of heavy holes or ions [6]. If the energy of electrons is scattered on acoustic phonons in semiconductors where the electron-phonon interaction has polarization character, or on certain types of atoms (H, He, for example) in gas plasmas, and the maximum temperature $T_{\max}$ of hot electrons is less than the energy of ionization, the energy relaxation time τ_ϵ can be assumed to be energy independent [16,17]. In other words, these cases can be effectively reduced to the case $p=s=0$ we have analyzed, because in the center of AS $T \gg T_0$. We noticed that the considered AS were experimentally discovered in semiconductors [10,11], where $T_{\max}$ is limited by impact ionization [8]. At the same time, such AS with gigantic values of $T_{\max}$ can form, for example, in the ionosphere heated by solar or UHF radiation [6,5].

Numerical simulations show that for sufficiently small α wide-peak AS is stable in some region of the values of $A > A_b$. For a given ϵ and α, at certain value of A we observed local breakdown in the AS center. In the case $\alpha \ll \epsilon$ this breakdown resulted in formation of a periodic sequence of wide-peak AS, and in the case $\alpha \gtrsim \epsilon$ wide-peak AS transformed into a pulsating one. With an increase of A this pulsating AS may split, and as a result a complex periodically or stochastically pulsating pattern, similar to the one observed in Ref. [9], forms in the system.

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